CIVIL-309: URBAN THERMODYNAMICS

Assist. Prof.
Dolaana Khovalyg

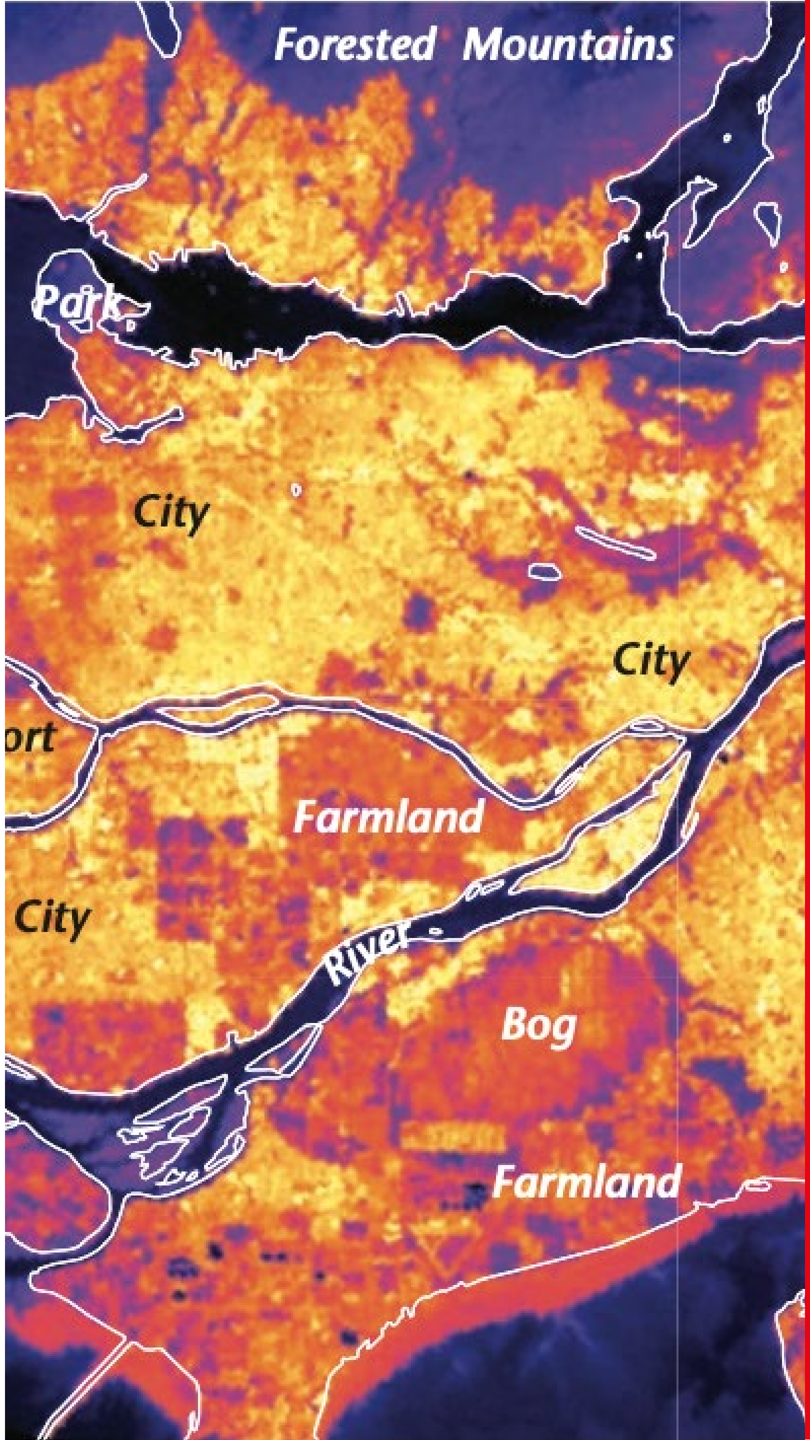
Lecture 04:

Heat Transfer and Thermal Properties: Convection and Evaporation

3 October 2025

Course Schedule

Week	Date	Time	Topics	Instructor
1	12.09	2 x 45’	**Course overview:** content, evaluation, topics **Urban characteristics, Urban Heat Island (UHI) effect**	DK
		1 x 45’	Introduction to the web tool **CityTherm** (part I)	DK
2	19.09	1 x 45’	**Overview of physical parameters**	DK
		1 x 45’	**Introduction to the course project I**	DK, JY
		1 x 45’	Supervised work on the course project I	JY
3	26.09	2 x 45’	**Heat Transfer: Conduction and radiation**	DK
		1 x 45’	Supervised work on the course project I	JY
4	03.10	2 x 45’	**Heat Transfer: Convection and evaporation**	DK
		1 x 45’	Supervised work on the course project I	JY
5	10.10	1 x 45’	**Campus walk:** exploring urban thermodynamics	DK, JY
		2 x 45’	Supervised work on the course project I	JY
6	17.10	3 x 45’	Supervised work on the course project I **Course project I submission deadline: 16:00 on October 17**	JY
7	24.10		**BREAK**	



CONTENT:

I. Convection

- Definitions
- Fluid properties
- Heat transfer coefficient
- Natural and forced convection

II. Combined modes of heat transfer

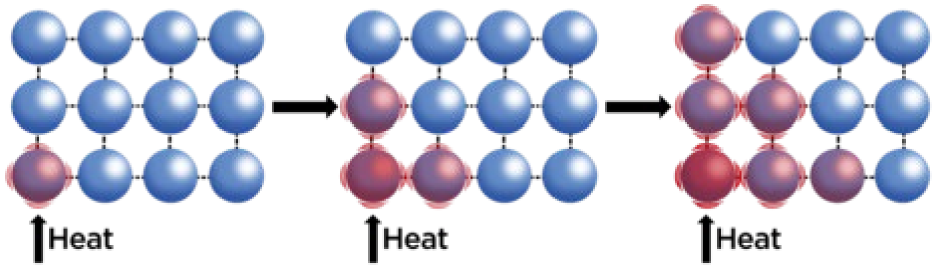
III. Evaporation

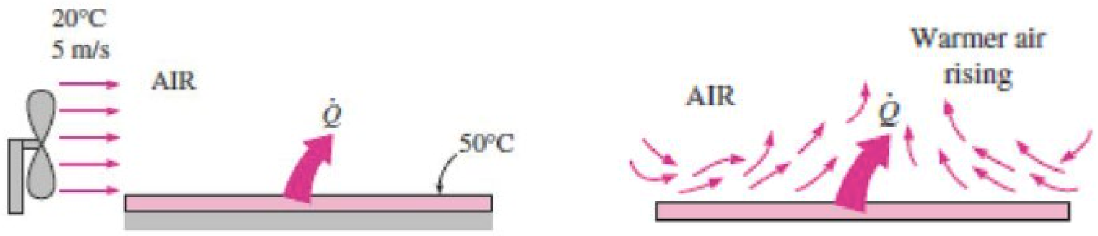
- Definitions
- Transport of water vapor
- Bowen ratio
- Methods to determine evaporation rate (Eddy Correlation, Bulk Transfer, Energy Balance, Penman)

Heat transfer: Mechanisms

Reminder from L3

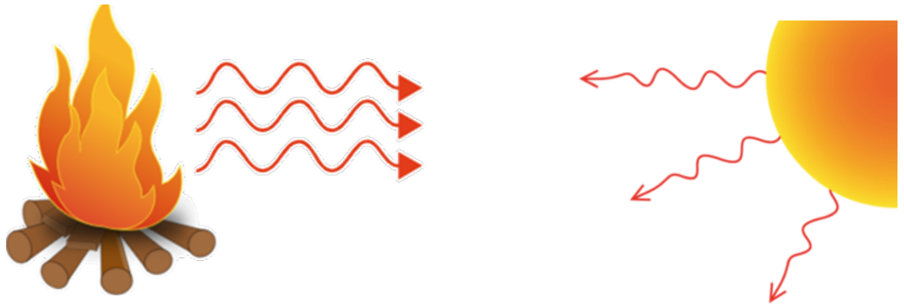
- **Sensible ("visible") heat transfer -** energy transferred as heat, require the existence of **a temperature difference.** There are **3 modes of heat transfer.**
  - **Conduction** – the transfer of thermal energy *from the more energetic particles* of a *substance to the adjacent less energetic ones* as a result of *interactions between particles.*
  - **Convection** – the transfer of thermal energy *between a solid surface* and *the adjacent liquid* or *gas* that is *in motion* (the faster the fluid motion, the greater the convection heat transfer).
  - **(Thermal) Radiation** – the energy emitted by matter in the form of *electromagnetic waves* (transfer at the speed of light).





(*a*) Forced convection

(*b*) Free convection



Which of the following dimensionless numbers in convection do you know about?

(choose up to 3 at the same time, or select separately ALL or NONE)

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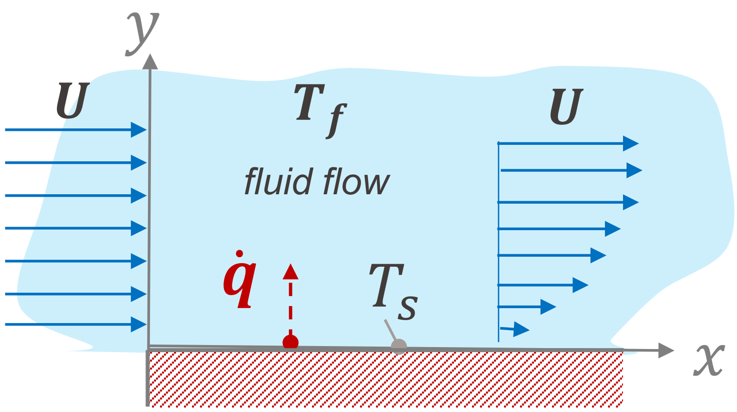
A. Nusselt number
B. Prandtl number
C. Rayleigh number
D. Reynolds number
E. Grashof number
F. All
G. None

Convection: Definitions

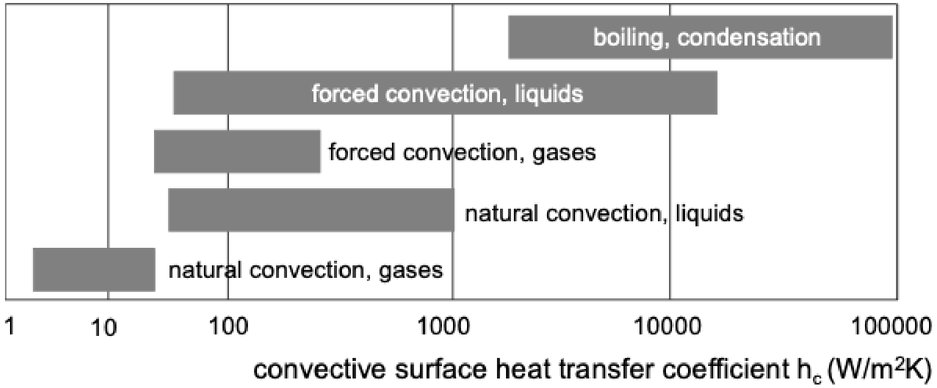
- **Transport of energy** due to **diffusion** (random collisions of molecules) and by **bulk motion of the fluid** (heat transfer from a surface to a moving fluid or between different fluids).
- The **rate of heat transfer** q $(\frac{W}{m^2})$ *from* or *to* a **surface** is *proportional* to **heat transfer coefficient** and **the temperature difference** between **the surface temperature** (T_s) and **the temperature of the free stream fluid** (T_f):

Newton's Law of Cooling: $q = h_{conv} \cdot (T_s - T_f)$ (4-1)



If $T_s > T_\infty$*, the convection heat flux is transferred* ***from*** *the surface*

Source: Medved, Building Physics, p. 22



- **Convective heat transfer coefficient** h_{conv} $(\frac{W}{m^2 \cdot K})$: **the rate of heat transfer** (in 1 sec) *between a solid surface* and a *fluid per* **unit surface area** (1 m²) *per* **unit temperature difference** (1 K).
- **Convective thermal resistance** R_{conv} $(\frac{m^2 \cdot K}{W})$: the *reciprocal* of convective heat transfer coefficient, indicates ability of the medium to resist a heat flow.

$$R_{conv} = \frac{1}{h_{conv}} \quad (4\text{-}2)$$

Convection: Heat Transfer Coefficient

- **Convective heat transfer coefficient h_{conv}** depends on:
  - **Fluid properties** $(\mu\ , k,\ \rho,\ c_p)$
  - **Solid surface characteristics** (its geometry and roughness)
  - **Intensity of fluid flow** (fluid velocity V)
  - **Fluid motion** (natural vs. forced)
  - **Type of the fluid flow** (laminar vs. turbulent)

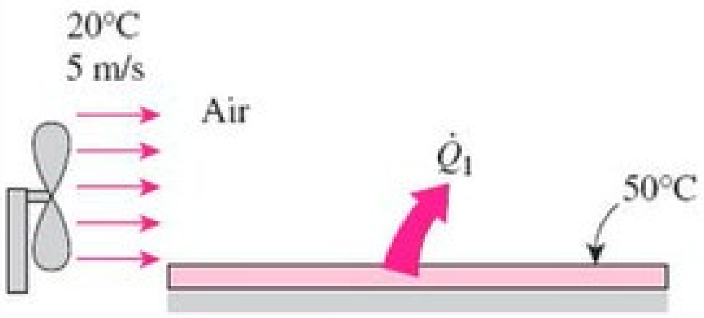
$$h_{conv} = \frac{Nu \cdot k}{L_c} \quad \text{(4-3a)}$$

$$Nu = \frac{\dot{q}_{conv}}{\dot{q}_{cond}} = \frac{h_{conv} \cdot \Delta T}{k \cdot \Delta T / L} = \frac{h_{conv} \cdot L_c}{k} \quad \text{(4-3b)}$$

L_c - characteristic length (m)

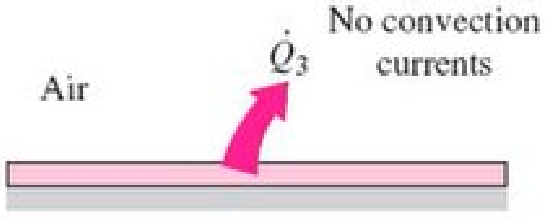
- **Nusselt number, Nu** $(-)$: ***dimensionless* convective heat transfer number**, represents *the enhancement of heat transfer* through a fluid layer as a result of *convection relative* to *conduction* across the same fluid layer.

  $Nu \approx 1$ - heat transfer across the layer by pure conduction, $Nu \gg 1$- more effective convection



(*a*) Forced convection

(*b*) Natural convection



(*c*) Conduction

Source: Cengel & Ghajar, Heat and Mass Transfer (5th edition), Fig. 6-1, p. 378

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Conduction Properties

Reminder from L3

- **Density, ρ** $(\frac{kg}{m^3})$: **mass** of **a unit volume** of a material substance
- **Thermal conductivity, k** or **λ** $(\frac{W}{m \cdot K})$: **the rate of heat transfer** (in 1 sec) through **a unit of thickness** (1 m) of the material **per unit area** (1 m²) per **unit temperature difference** (1 K/m).

$$\boldsymbol{k} = \alpha \cdot C_p \quad (3\text{-}11)$$

- **Thermal diffusivity, α** $(\frac{m^2}{s})$: **time** at which temperature *change travels* and **depth** of the layer involv*ed in thermal changes,* represents how fast heat diffuses through a material. A material with low thermal diffusivity, such as sand, doesn't store much of the thermal energy.

$$\boldsymbol{\alpha} = \frac{k}{\rho \cdot c_p} \quad (3\text{-}12)$$

- **Specific heat capacity (isobaric), c_p** $(\frac{J}{kg \cdot K})$: ability of a material to store heat. Energy absorbed by 1 kg of material when its temperature increases by 1 K, highly dependent on temperature.
- **Heat capacity (isobaric), C_p** $(\frac{J}{m^3 \cdot K})$: specific heat capacity expressed in terms of the volume.

$$\boldsymbol{C_p} = \rho \cdot c_p \quad (3\text{-}13)$$

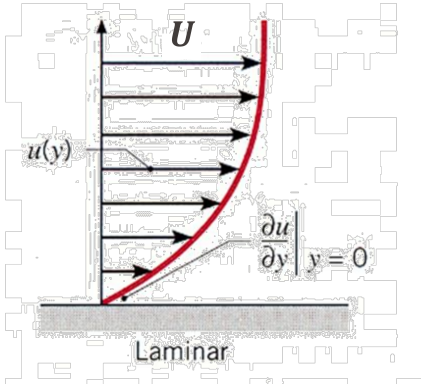
Fluid Properties: Conductive and Convective

		Both	Conduction			Convection		
Fluid Type	$t\ (^{o}C)$	$\rho\ (\frac{kg}{m^3})$	$k\ (\frac{W}{m \cdot K})$	$c_p\ (\frac{kJ}{kg \cdot K})$	$\alpha \cdot 10^{-6}\ (\frac{m^2}{s})$	$\mu \cdot 10^{-6}\ (\frac{N \cdot s}{m^2})$	$v \cdot 10^{-6} (\frac{m^2}{s})$	Pr
Air	-10	1.334	0.023	1.006	17.62	16.61	12.6	0.717
	0	1.287	0.024	1.006	18.94	17.11	13.49	0.714
	20	1.194	0.026	1.007	21.58	18.11	15.27	0.709
	40	1.118	0.027	1.008	24.42	19.07	17.2	0.705
	60	1.052	0.029	1.008	27.38	20.02	19.21	0.702
Water	0	1000	0.555	4.219	0.13	1790	1.79	13.61
	20	998	0.598	4.182	0.14	1008	1.01	7.063
	40	992	0.627	4.178	0.15	653	0.66	4.385
	60	983	0.651	4.19	0.16	470	0.48	3.077
Argon	0	1.784	0.018	0.52	19.09	20.99	11.77	0.616
Krypton	0	3.75	0.0094	0.248	10.14	23.29	6.21	0.613
Xenon	0	5.9	0.0056	0.158	6.05	21.1	3.58	0.591

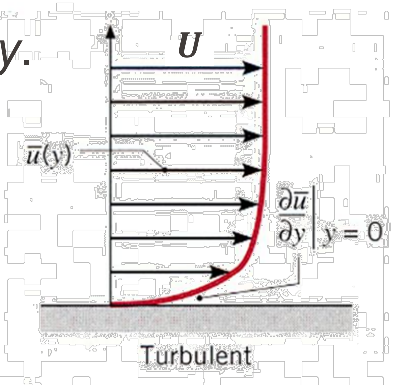
Source: Medved, Building Physics, p. 19

Convection Properties

- **Viscosity** is the material property which relates **the viscous stresses** in a material to **the rate of change** of **a deformation** (the strain rate).
- **Dynamic viscosity** $\boldsymbol{\mu}$ ($\frac{N\cdot s}{m^2}$ or $\frac{kg}{m\cdot s}$): *proportionnality constant* between **the vertical gradient of horizontal velocity** ($\partial u/\partial y$) and **the induced shear stress** (τ_w). It characterizes the *resistance to deformation* at *a given rate*.



- **Kinematic viscosity** $\boldsymbol{v}$ ($\frac{m^2}{s}$): the ratio of the *dynamic viscosity* to the *fluid density*.



$$\tau_w = \boldsymbol{\mu} \cdot \left(\frac{\partial u}{\partial y}\right)_{y=0} \quad (4\text{-}4)$$

$$\boldsymbol{v} = \frac{\mu}{\rho} \quad (4\text{-}5)$$

Material properties can be combined into dimensionless numbers:

- **Prandtl number,** $\boldsymbol{Pr}$ ($-$): the *relative thickness* of the **velocity** and the **thermal boundary layers**, the ratio of **momentum diffusivity** to **thermal diffusivity** ($\boldsymbol{Pr \approx 1}$ in gases, $\boldsymbol{Pr \approx 10}$ for water).

$$\boldsymbol{Pr} = \frac{Molecular\ diffusivity\ of\ momentum}{Molecular\ diffusivity\ of\ heat} \quad (4\text{-}6)$$

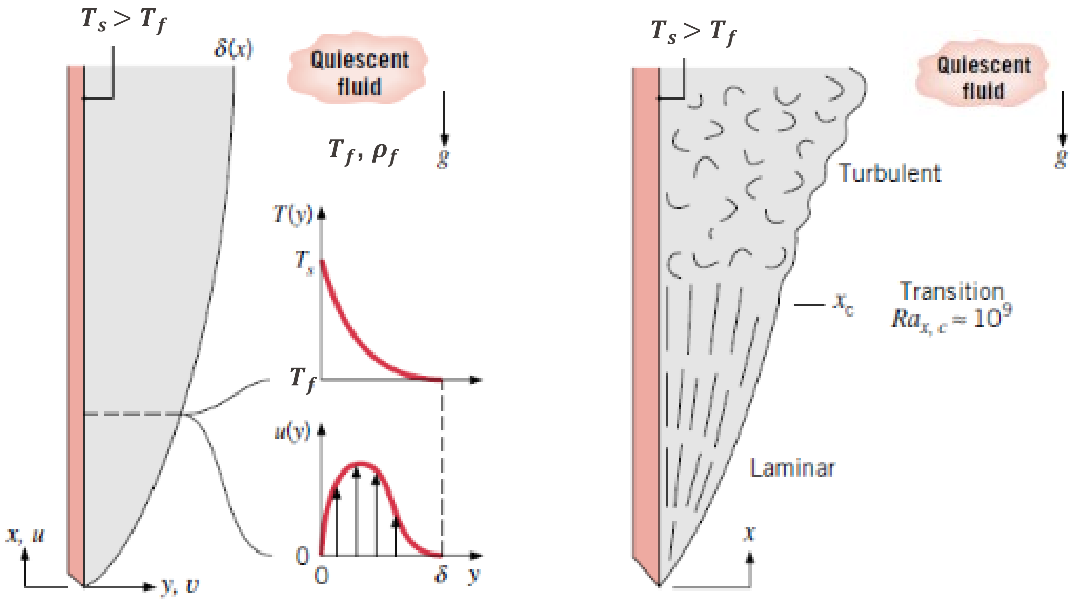
$$\boldsymbol{Pr} = \frac{v}{\alpha} \quad (4\text{-}6a)$$

$$\boldsymbol{Pr} = \frac{c_p \cdot \mu}{k} \quad (4\text{-}6b)$$

Convection: Natural (Free)

- **Natural** or **free convection**: **motion** of **fluid** *resulting* from a **temperature gradient** (the fluid motion is due to the *difference in density* induced by *the difference in temperature*). The driving force of natural convection is **buoyancy**.

- **Volumetric thermal expansion coefficient** $\boldsymbol{\beta}$ $(\frac{1}{K})$: expresses *the net buoyancy force*, the **variation of the density** of a fluid **with temperature** at **constant pressure**.



$$\boldsymbol{\beta} = -\frac{1}{\rho}\cdot\left(\frac{\partial\rho}{\partial T}\right)_P \quad \text{(4-7a)} \quad \xrightarrow[\textit{for ideal gas}]{} \quad \boldsymbol{\beta} = \frac{1}{T} \quad \text{(4-7b)}$$

$$T = \frac{T_s + T_f}{2} \quad \text{(4-8)}$$

Flow regime for vertical flat plates:

- Laminar flow $\boldsymbol{Ra} \le 10^9$
- Turbulent flow $\boldsymbol{Ra} > 10^9$

- **Grashof number $\boldsymbol{Gr}$** (-): the ratio of **the buoyancy force** to **the visocus force** acting on the fluid, governs the flow regime in natural convection.

$$\boldsymbol{Gr} = \frac{Boyancy}{Viscosity} \qquad \boldsymbol{Gr} = \frac{g\cdot\beta\cdot(T_s - T_f)\cdot L_c^3}{\nu^2} \quad \text{(4-9)}$$

for $T_s > T_f$

- **Rayleigh number $\boldsymbol{Ra}$** (-): the product of $\boldsymbol{Gr}$ and $\boldsymbol{Pr}$ numbers, the ratio of *buoyancy force and thermal* and *momentum diffusivities*.

$$\boldsymbol{Ra} = Gr \cdot Pr \quad \text{(4-10)}$$

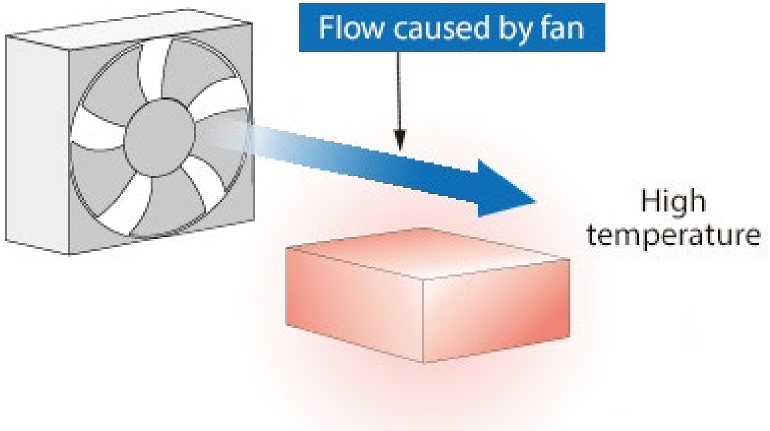
Source: Cengel & Ghajar, Heat and Mass Transfer (5th edition), p. 531-545

Convection: Forced

- **Forced convection**: **fluid motion** *forced* by an **external source** (i.e., wind, fan).
- **Reynolds number $\boldsymbol{Re}$** (-): the ratio of **inertial forces** to **viscous forces** within a fluid.
    - At small $\boldsymbol{Re}$, the *viscous forces* are large enough *to suppress* random fluctuations → **laminar flow**
    - At large $\boldsymbol{Re}$, the *inertial forces* ($\sim \rho, V$) are *large relative* to viscous forces, thus, the viscous forces *can not prevent* the random and rapid fluctuations of the fluid → **turbulent flow**

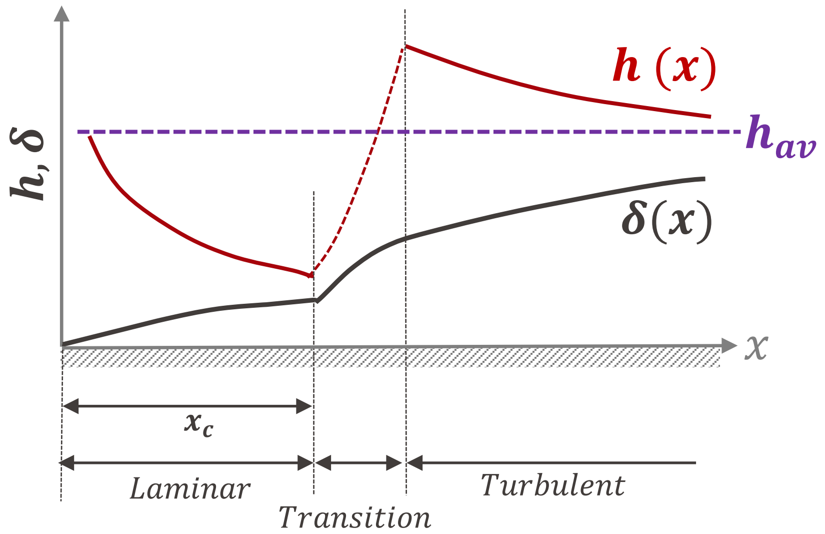
$$Re = \frac{Intertia\ forces}{Viscous\ forces} \quad \text{(4-11)}$$

$$Re = \frac{U \cdot L_c}{\nu} \quad \text{(4-11a)} \qquad Re = \frac{\rho \cdot U \cdot L_c}{\mu} \quad \text{(4-11b)}$$



Flow regime for horizontal flat plates:

- Laminar flow $\boldsymbol{Re} \leq 5 \cdot 10^5$
- Turbulent flow $\boldsymbol{Re} > 5 \cdot 10^5$



Overview of dimensionless numbers in convection:

- **Nusselt number, Nu**

$$\boldsymbol{Nu} = \frac{Convection}{Conduction}$$

(4-3b)
$$\boldsymbol{Nu} = \frac{\boldsymbol{h_{conv}} \cdot L_c}{k}$$

- **Prandtl number, Pr**

$$\boldsymbol{Pr} = \frac{Diffusivity\ of\ momentum}{Diffusivity\ of\ heat}$$

(4-6a)
$$\boldsymbol{Pr} = \frac{\nu}{\alpha}$$

(4-6b)
$$\boldsymbol{Pr} = \frac{c_p \cdot \mu}{k}$$

- **Rayleigh number, Ra**

$$\boldsymbol{Ra} = Boyancy\ force \cdot [Diffusivity\ of\ momentum\ \&\ heat]$$

(4-10)
$$\boldsymbol{Ra} = Gr \cdot Pr$$

- **Grashof number, Gr**

$$\boldsymbol{Gr} = \frac{Boyancy\ force}{Viscous\ forces}$$

(4-9)
$$\boldsymbol{Gr} = \frac{g \cdot \beta \cdot (T_s - T_f) \cdot L_c^3}{\nu^2}$$

$for\ T_s > T_f$

- **Reynolds number, Re**

(4-11)
$$\boldsymbol{Re} = \frac{Intertia\ forces}{Viscous\ forces}$$

(4-11a)
$$Re = \frac{U \cdot L_c}{\nu}$$

(4-11b)
$$Re = \frac{\rho \cdot U \cdot L_c}{\mu}$$

Which of the following dimensionless numbers related to convection do you understand well now?

(choose up to 3 at the same time, or select separately ALL or NONE)

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A. Nusselt number
B. Prandtl number
C. Rayleigh number
D. Reynolds number
E. Grashof number
F. ALL
G. NONE

Which of the following parameters characterize pure convection?

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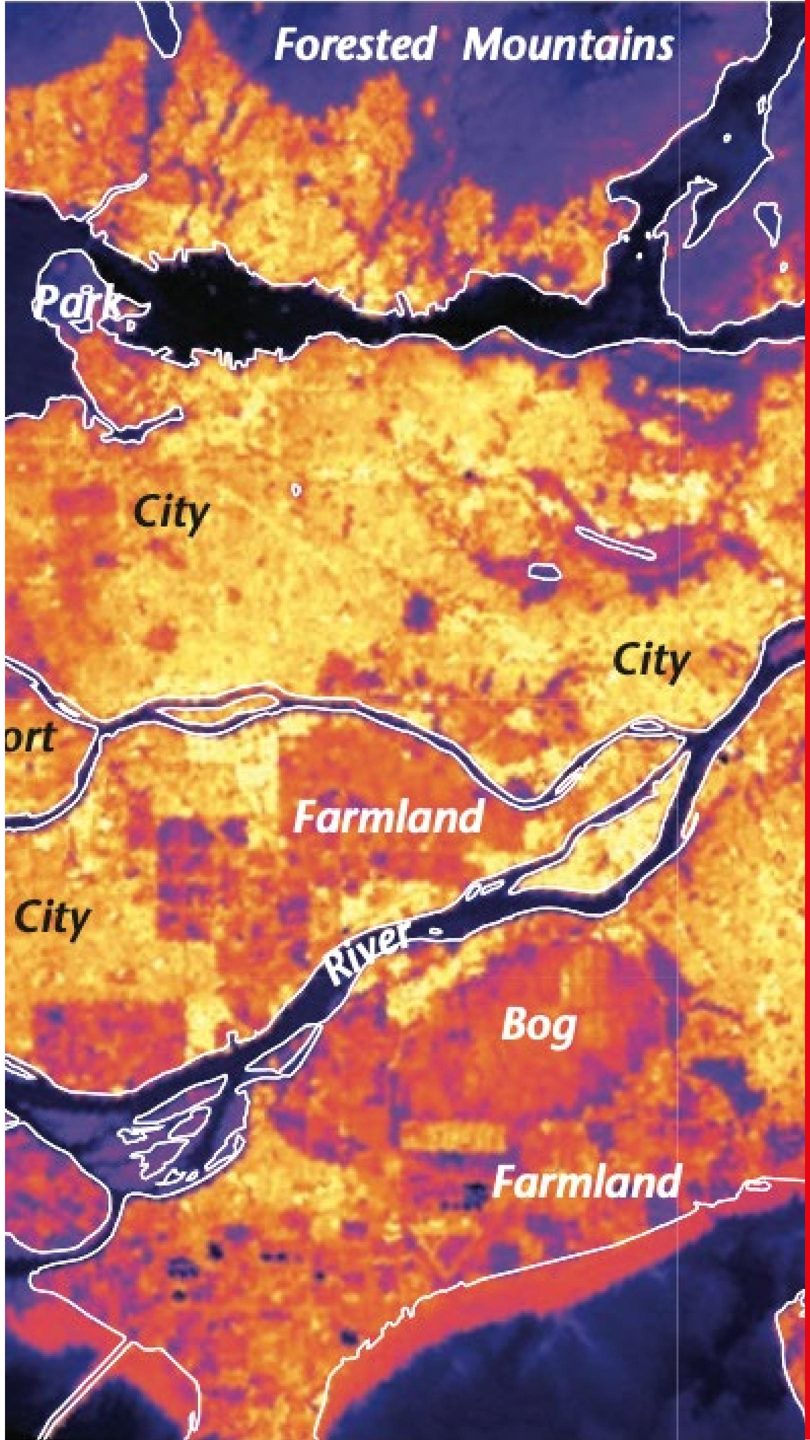
A. Specific heat capacity
B. Dynamic viscosity
C. Thermal diffusivity
D. Thermal conductivity

Convective Heat Transfer: External Fluid Flow

- **Natural convection**: $Nu = C \cdot Ra^n$ (4-12)
- **Forced convection**: $Nu = C \cdot Re^m \cdot Pr^n$ (4-13)

Nu number (dimensionless heat transfer coefficient) is typically determined *experimentally* or using *numerical methods.*

Mode	Surface Type	Flow Type	C	m	n	Validity Range	Characteristic dimension L_c
Free	Vertical plate	Laminar	0.59	-	1/4	$Ra_H < 10^9$	Plate height *H*
		Turbulent	0.10	-	1/3	$Ra_H > 10^9$	Plate height *H*
	Upper surface of a heated horiz. plate / lower surface of a cooled horiz. plate	Laminar	0.54	-	1/4	$Ra_L\text{: } 10^4 - 10^7$	$Area/Perimeter$
		Transition	0.15	-	1/3	$Ra_L\text{: } 10^7 - 10^{11}$	$Area/Perimeter$
	Lower surface of a heated horiz. plate / upper surface of a cooled horiz. plate	All	0.27	-	1/4	$Ra_L\text{: } 10^5 - 10^{11}$	$Area/Perimeter$
Forced	Horizontal flat plate	Laminar	0.664	0.5	1/3	$Re_L < 5 \cdot 10^5$	Plate length L
		Turbulent	0.037	0.8	1/3	$Re_L\text{: } 5 \cdot 10^5 - 10^7$ $Pr\text{: } 0.6 - 60$	Plate length L



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Combined modes: Thermal Resistance and Transmittance

- **Heat flux $\dot{q}$ ($\frac{W}{m^2}$)** formulation:

$$\dot{q} = U_{tot} \cdot \Delta T = \frac{1}{R_{tot}} \cdot \Delta T \quad (3\text{-}8)$$

$$U = \frac{1}{R} \quad (3\text{-}9)$$

Thermal transmittance,
Overall heat transfer coefficient,
[W/m²K]

*Temperature gradient, [K] **

- **Thermal Transmittance U** (U-value, $\frac{W}{m^2 \cdot K}$): heat transfer coefficient, an **indicator of the efficiency** to **promote** heat conduction by the material.
- **Thermal Resistance R** (R-value, $\frac{m^2 \cdot K}{W}$): the capacity of a material to ***resist*** **heat flow.**

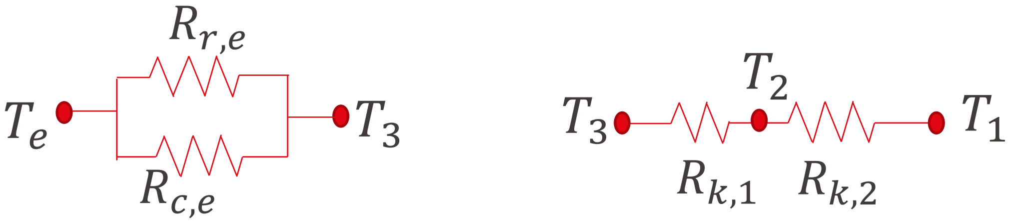
- **Conduction**: $\dot{q}_{cond} = k\frac{\Delta T}{L}$ (3-5b) $\Rightarrow R_{cond,i} = \frac{L_i}{k_i}$ (3-10)
- **Convection**: $\dot{q}_{conv} = h_{conv}\,(T_s - T_f)$ (4-1) $\Rightarrow R_{conv} = \frac{1}{h_{conv}}$ (4-2)
- **Radiation**: $\dot{q}_{rad} = h_{rad}\,(T_s - T_\infty)$ (3-18b) $\Rightarrow R_{rad} = \frac{1}{h_{rad}}$ (3-23)

1-D formulations

* *Temperature gradient should be positive since heat flows **spontaneously** from the hot side to the cold side according to the 2nd law of thermodynamics*

Combined modes: Thermal Resistance

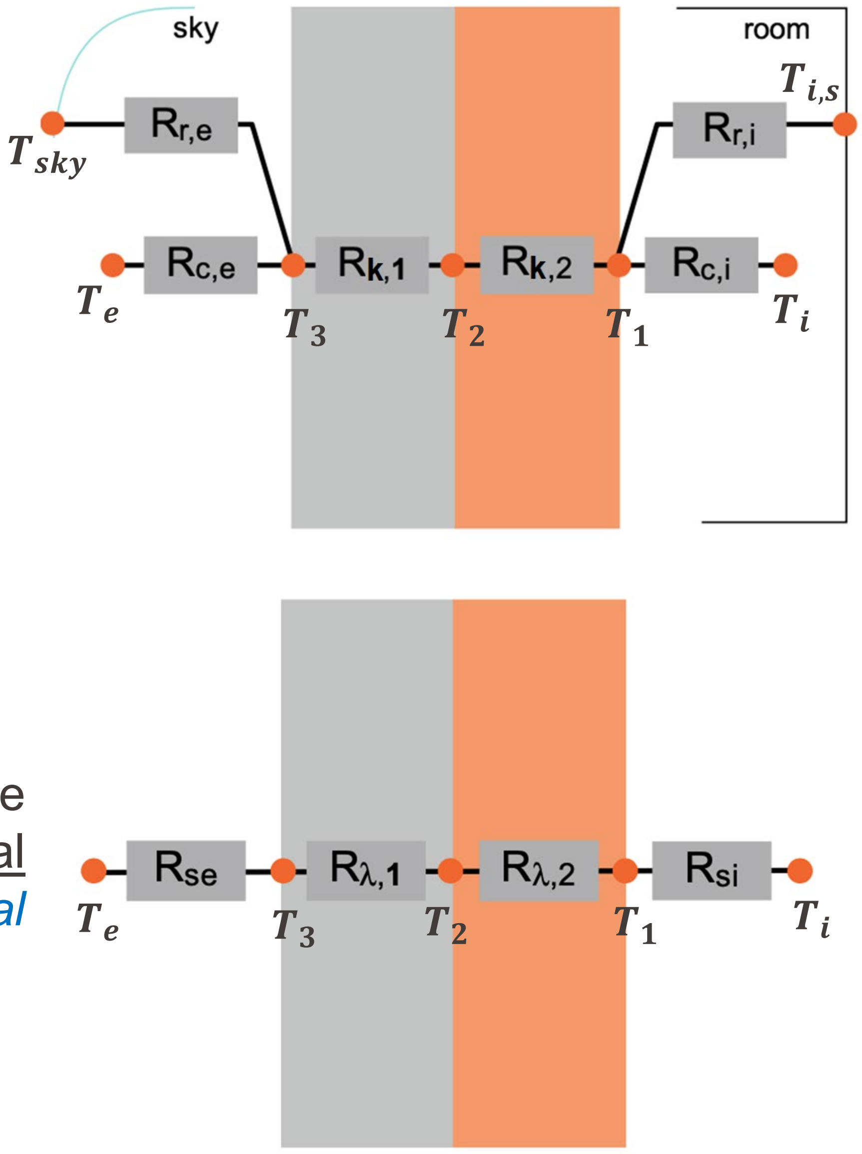
- Using *the electrical circuit analogy*, heat transfer problems *involving multiple modes of heat transfer* can be analyzed using **the network of thermal resistances** forming a thermal circuit. Considering Eqn. (3-6)-(3-7) from Lect. 3:



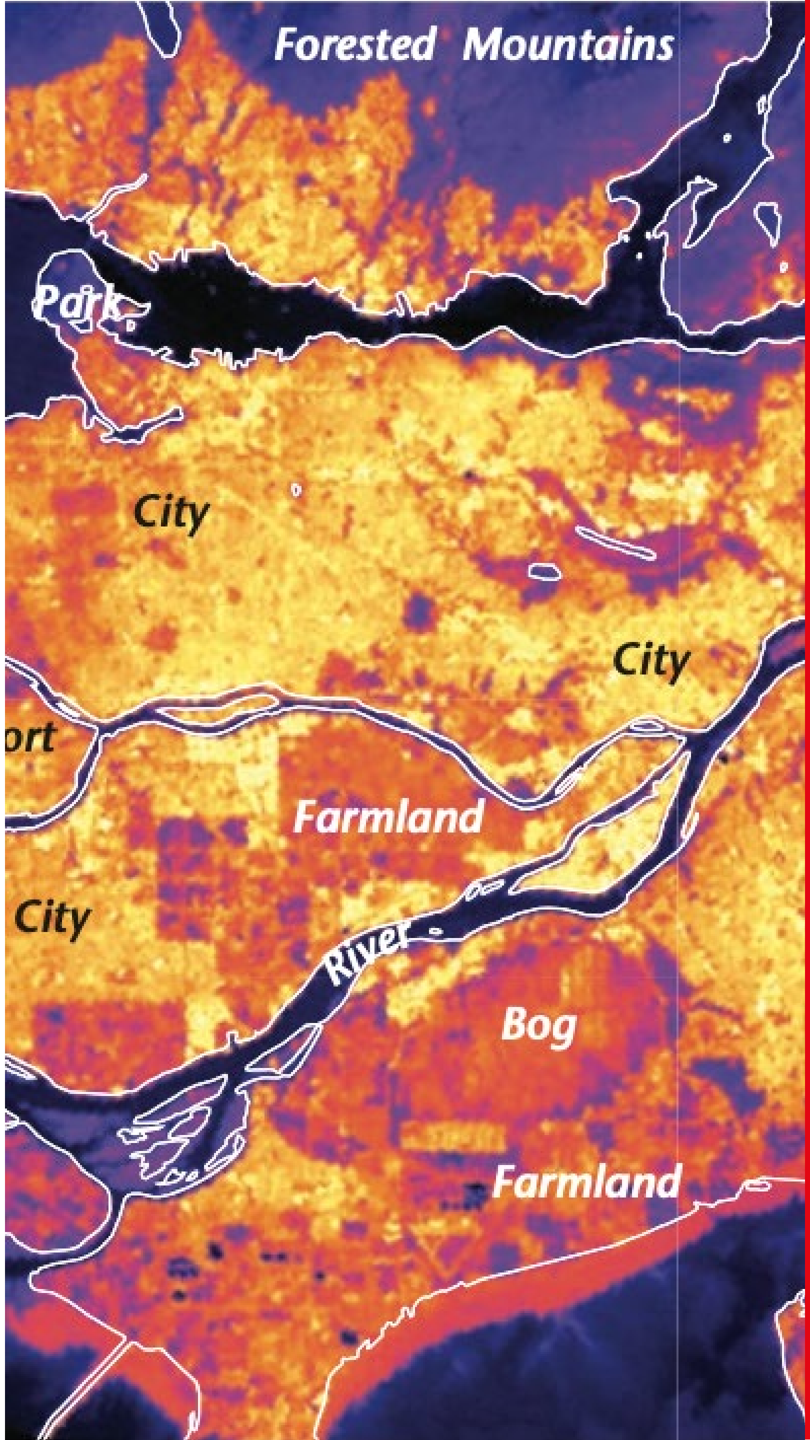
$$R_{tot} = \underbrace{\frac{1}{\frac{1}{R_{r,e}}+\frac{1}{R_{c,e}}}}_{R_{se}} + R_{k,1} + R_{k,2} + \underbrace{\frac{1}{\frac{1}{R_{r,i}}+\frac{1}{R_{c,i}}}}_{R_{si}} \qquad (4\text{-}14)$$

- For buildings, the convective and radiative resistances of the outer and inner wall are *combined* in a <u>single thermal resistance</u> R_{se} or R_{si} (referred as a *surface thermal resistance*). For a wall with j number of solid layers:

$$U = \frac{1}{R_{tot}} = \frac{1}{R_{si} + \sum_{j=1}^{n}\frac{L_j}{k_j} + R_{se}} \qquad (4\text{-}15)$$



Source: Medved, Building Physics, p. 41



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Urban physical processes: Urban water

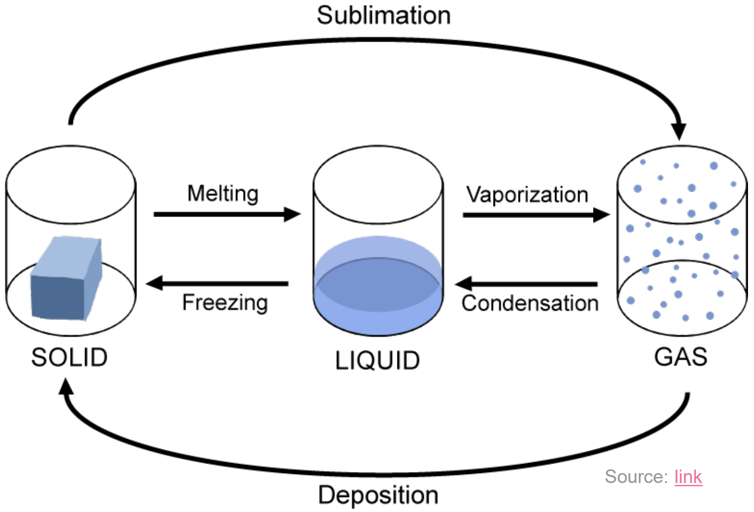
- The urban water availability is of interest for its capacity to **evaporate**. Only *liquid* water evaporates. Heat can be used as **latent heat** instead of sensible heat (temperature of water does not increase, only enthalpy changes)
- During evaporation, the energy is stored as latent heat in the state of gaseous water and will be released when water condensate. Condensation is also named **dewfall**.

- **Latent heat flux density Q_E** ($\frac{W}{m^2}$): energy flux that is used to evaporate the water mass.

$$\boldsymbol{Q_E} = L_v \cdot \boldsymbol{E} \qquad (1\text{-}6)$$

L_v - **latent heat of vaporization** ($2.464\ \frac{MJ}{kg}$ at 15 °C)

E - **mass flux density of water** in $\frac{kg}{m^2 \cdot s}$.



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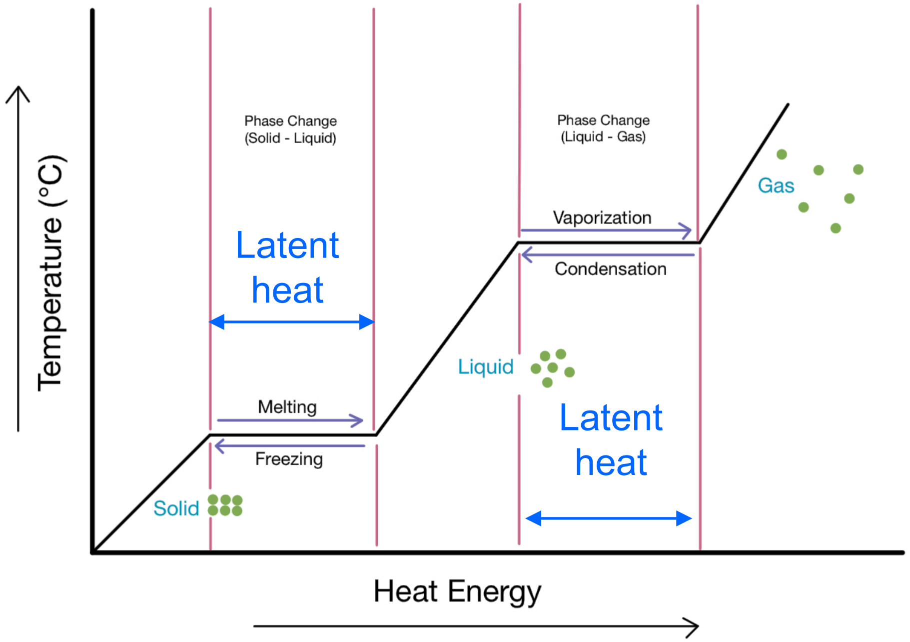


Source: https://www.expii.com/t/phase-change-diagrams-overview-examples-8057

Evaporation: Definitions

- **Evaporation**: *phenomenon* by which a substance is converted from the **liquid state** into **vapor state** (the reverse process of evaporation is **condensation**).
- **Atmospheric evaporation** takes place from:
  - Free water surface
  - Moist surfaces (soil, artificial surfaces)
  - Leaves of living plants and trees
- **The amount of water evaporated** depends on:
  - the **supply of energy to release latent heat**
  - the physical process of **transport of vapor away from any interface**
  - …and **the supply of water.**
- Vapor is transferred almost the same way as *heat* and *momentum*, by *advection* and *pressure gradients.*

Images from weblink

Source: Arya, Introduction to micrometeorology, Chapter 12, p. 242

Evaporation: **Definitions**

Note: $1\, kg/(m^2 \cdot s) = 1\, mm/s$

- **Potential evaporation** $\mathrm{E_{pot}}$, $kg/(m^2 \cdot s)$:

  the maximum rate of *evaporation* for a given surface conditions with no water supply limitation. **Free evaporation** depends on the prevailing meteorological conditions

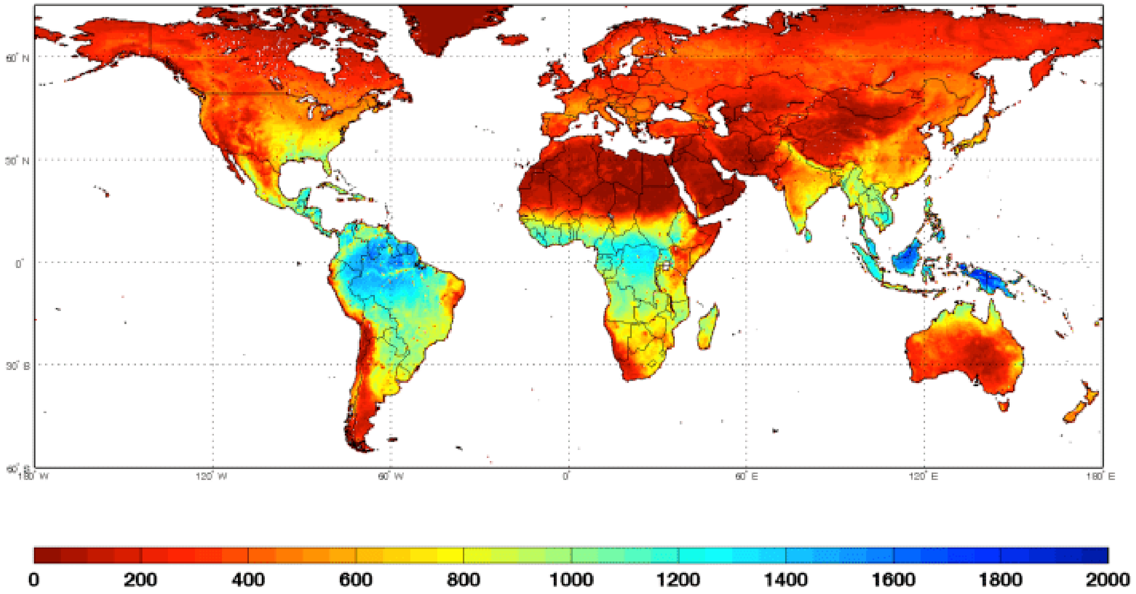
- **Actual evaporation** E, $kg/(m^2 \cdot s)$:

  always **less** than *potential evaporation*. It is **most often less** than potential evaporation because the surface may not be saturated

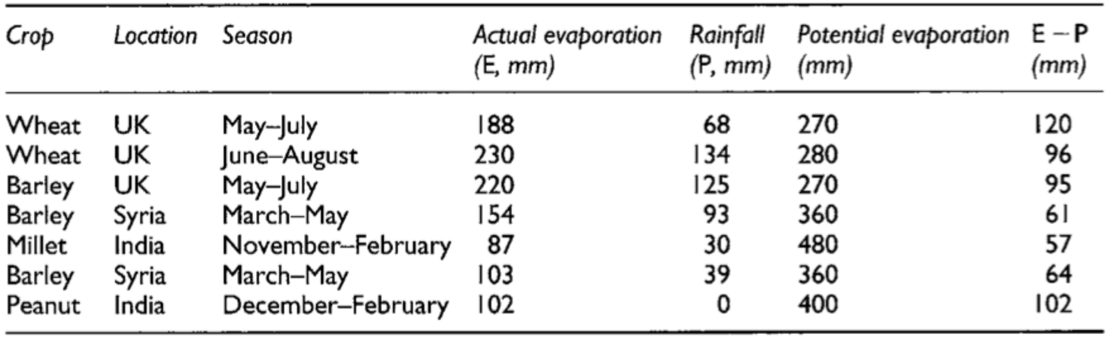
$$E < E_{pot} \quad (4\text{-}16)$$

- If a surface is not saturated with water, **the rate of evaporation** depends *on its moisture content*
- **Potential evaporation** for soil, vegetation or an artificial surface is generally *less* than the *free water surface evaporation* under *the same weather conditions*, especially in humid regions.

World’s potential evapotranspiration (mm/y), 2006



Source: Wang et al. (2012) 10.5194/hessd-9-4777-2012



Crop	*Location*	*Season*	*Actual evaporation (E, mm)*	*Rainfall (P, mm)*	*Potential evaporation (mm)*	*E – P (mm)*
Wheat	UK	May–July	188	68	270	120
Wheat	UK	June–August	230	134	280	96
Barley	UK	May–July	220	125	270	95
Barley	Syria	March–May	154	93	360	61
Millet	India	November–February	87	30	480	57
Barley	Syria	March–May	103	39	360	64
Peanut	India	December–February	102	0	400	102

Source: Mason, Introduction to environmental physics, p. 327

Evaporation rate (E): Transport of Water Vapor

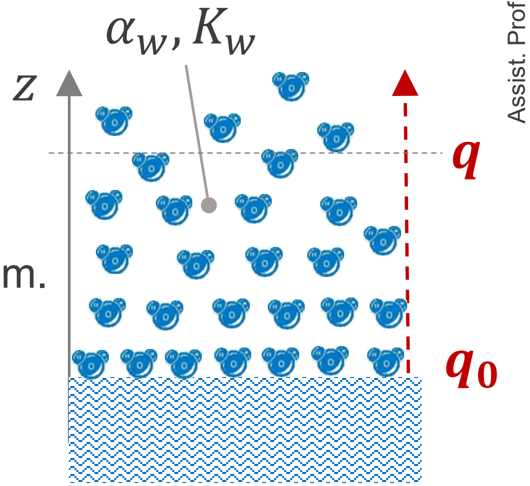
- The expression of the **evaporation rate** $\boldsymbol{E}$ $(\frac{kg}{m^2 s})$ due to **the transport of water vapor** over a **surface** depends whether the **flow** is **laminar** or **turbulent**.

- **Laminar flow: Fick's law** for the evaporation rate at *a horizontal surface*:

$$E = -\rho \cdot \boldsymbol{\alpha_w} \cdot \frac{\partial q}{\partial z} \quad (4\text{-}17)$$

$\boldsymbol{\alpha_w}$ (m^2/s) - molecular diffusivity of water vapor
$\boldsymbol{q}$ (kg/kg) - specific humidity (*see L2, slide 24*)

  - ○ Only valid when molecular exchange is the primary (perhaps the only) transport mechanism.
  - ○ Fick's law is not applicable for *urban areas* because the flow in the atmosphere at *the surface layer* at local scale is *always turbulent.*



- **Turbulent flow:** the evaporation rate at *a horizontal surface* is given by **the eddy correlation method**:

1 D, steady state

$$E = -\rho \cdot \boldsymbol{K_w} \cdot \frac{\partial q}{\partial z} \quad (4\text{-}18)$$

$\boldsymbol{K_w}$ (m^2/s) - eddy diffusivity

  - ○ Turbulent fluxes arise due to *the existence of a gradient of a related property in the air* and *turbulent motion* set off either by *mechanical* or *thermal* production. The greater the gradient and the more efficient the mixing, the stronger the flux density.
  - ○ The intensity of turbulent mixing depends on the *surface roughness*, *wind shear* or *friction velocity*, and *thermal stratification*. The evaporation rate also depends on the above factors, plus *the average specific humidity gradient*.

Evaporation rate (E): Numerical Methods

- Many different **numerical methods** have been developed **to assess evaporation rates**:

1. **Eddy correlation** or **covariance method:** expresses the evaporation heat flux as a function of *the humidity gradient*.
2. **Bulk transfer approach:** expresses the evaporation rate as a *linear* function of *specific humidity* with *the bulk coefficient*.
3. **Energy balance** or **Bowen ratio method:** estimates the evaporation with the expressions of the *energy balance* and *the Bowen ratio*.
4. **Penman approach**: a synthesis of *the energy balance* and *the bulk transfer method*.
5. **Gradient** or **Aerodynamic method:** under neutral stability conditions, both the wind speed and specific humidity follow logarithmic profile laws.
6. **Profile method:** *evaporative heat flux profile* over urban areas *is known* and expressed as *a function of the altitude*.

- The **energy balance method** assesses evaporation *due to an external supply of energy converted to latent heat* E_S while **the eddy correlation method, the bulk transfer approach** assesses the evaporation *due to the transport of water vapor* E_T.

- **The Penman approach** is *the most used* and has numerous applications in soil and vegetation evaporation assessment.

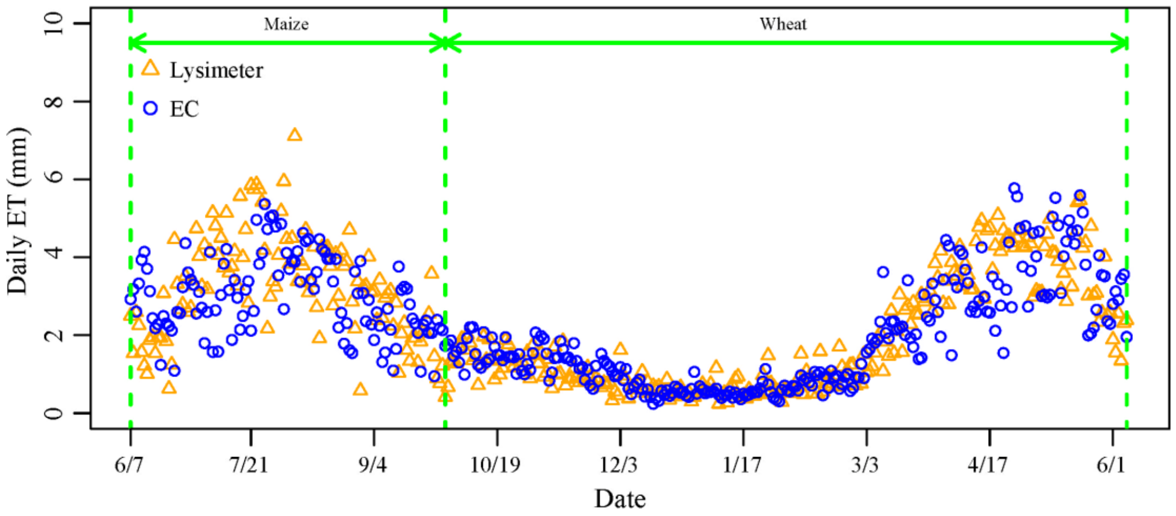
- The **gradient** and **profile methods** give *empirical expressions* of the evaporation function of the altitude $E = f(z)$.

Evaporation rate (E): Eddy Correlation Method

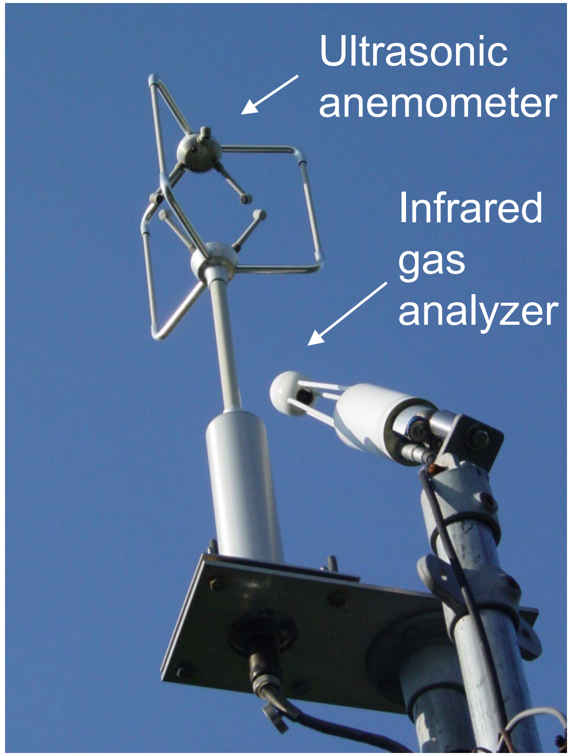
- The **eddy correlation method** is used to *directly* measure **the local water vapor flux E $\left(\frac{kg}{m^2 s}\right)$** over a *homogeneous* or *non-homogeneous* surface.
- **It requires** measurements fluctuations of vertical velocity w and specific humidity q, and calculation of their covariance $\overline{w \cdot q}$.
- *Difficult* to obtain input parameters w and q, as the fast-responding instruments are required, adequately calibrated and installed with care.
- The method is **the most accurate one**, used to *calibrate* other methods such as **aerodynamic**, **Bowen** and **Penman** methods.

$$E = \rho \cdot \overline{w \cdot q} \qquad (4\text{-}19)$$

$w\ (m/s)$ - vertical velocity
$q\ (kg/kg)$ - specific humidity



Source: Wang et al. (2020) doi.org/10.1139/cjss-2020-0050



Source: https://en.wikipedia.org/wiki/Eddy_covariance

Evaporation rate: Bulk Transfer Method

- **A bulk transfer law** is the relationship between *surface kinematic flux of a meteorological variable* to *the product of wind speed* times *the difference of that variable between the surface and some reference height*.
- The **bulk transfer coefficient C_W** is **an empirical constant of proportionality** in a bulk law transfer.
- The **bulk transfer method** expresses the evaporation rate as:

$$E = \rho \cdot C_W \cdot U \cdot (q_0 - q_r) \quad \text{(4-20a)}$$

Can be simplified assuming that *specific humidity very close to the surface* equals *the one at the surface* $q_0 = q_s$:

$$E = \rho \cdot C_W \cdot U \cdot (q_s - q_r) \quad \text{(4-20b)}$$

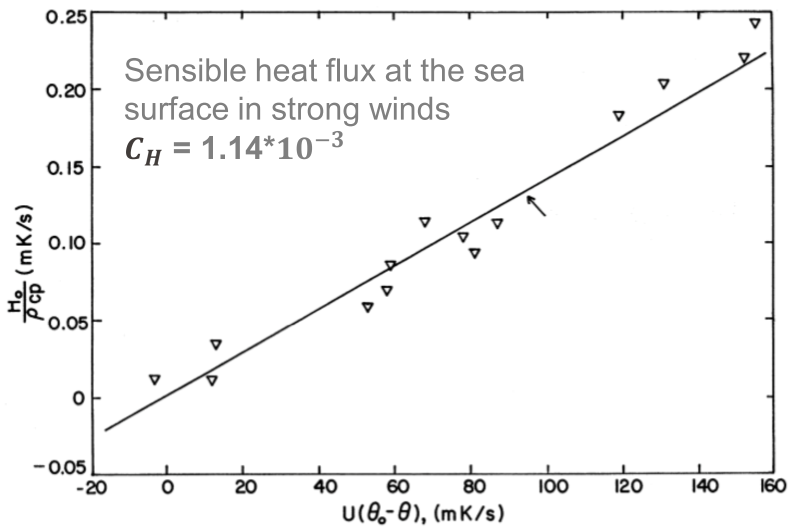
C_W (-) - bulk transfer coefficient for water vapor, ρ (kg/m^3) – air density,
U $(\frac{m}{s})$ – air speed at reference height, Subscripts: “0” (very close to the surface),
"s" (at the surface, saturated), “r” (reference height above the surface)

- The formulation is similar to the sensible heat flux, by Reynold’s analogy between *heat* and *mass* transfer:

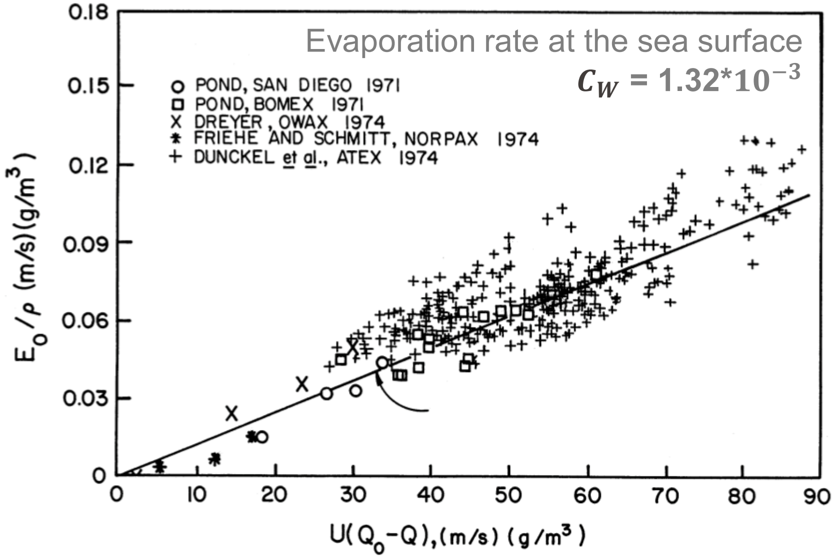
$$Q_H = \rho \cdot c_p \cdot C_H \cdot U \cdot (T_s - T_r) \quad \text{(4-21)}$$

C_H - bulk heat transfer coefficient, $C_W = C_H$ in Reynold’s analogy between water vapor and heat transfers

Source: Arya, Introduction to micrometeorology, p.252



Sensible heat flux at the sea surface in strong winds
C_H = 1.14*10^{-3}

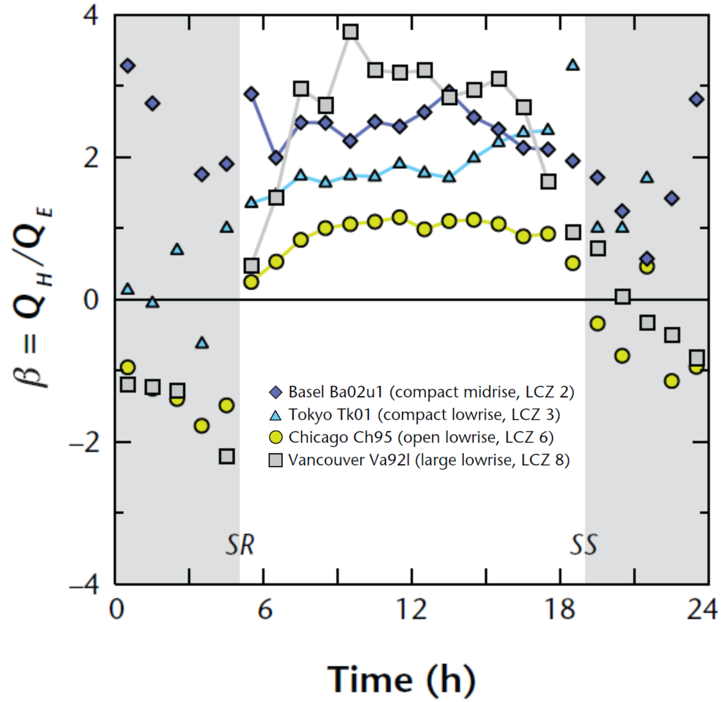


Evaporation rate at the sea surface
C_W = 1.32*10^{-3}

Evaporation: **Bowen Ratio**

- **Bowen ratio B** (-): the ratio of **sensible heat** to **latent heat flux**.

$$B = \frac{Q_H}{Q_E} \quad (4\text{-}22)$$



Typical daytime values of Bowen ratio for **natural** and **urban** systems (in bold – mean values, the range depends on moisture availability)

Surface description	LCZ	Bowen ratio B
Tropical oceans	G	0.1
Lakes	G	0.2 – 1
Crops	D	**0.3** 0.1 – 1
Urban 35 – 75% greenspace	6, 9	**1** 0.5 – 2.5
Urban 25 – 40% greenspace	3, 5	**2** 1.5 – 3
Urban < 20% greenspace	2, 3, 8	**4** 3 – 8
Semi-arid lands	C, F	2 – 6
Sandy desert	F	≈10

Source: Oke, Urban climates, p. 184

- The Bowen ratio informs regarding the *condition of* surfaces and near them:
  - $B > 1$ indicates that **the surface channels *more heat*** into **sensible form**. The *lower* atmosphere *gets warmer*.
  - $B < 1$ indicates that **latent heat dominates**. The *surface* and *near-surface air* is *cooler*, whilst *the surface adds humidity* to the *environment*.

Evaporation: Energy Balance Method

- **The Bowen ratio** can be estimated from **the gradient transport** relations for **sensible** and **latent heat** (with the assumption that *the eddy exchange coefficients* $K_h = K_w$ are equal):

$$B = \frac{c_p}{L_v} \cdot \frac{\frac{\partial T}{\partial z}}{\frac{\partial q}{\partial z}} = \frac{c_p}{L_v} \cdot \frac{\Delta T}{\Delta q} \approx \frac{c_p \cdot p_a}{0.622 \cdot L_v} \cdot \frac{\Delta T}{\Delta p_v} = \boldsymbol{\gamma} \cdot \frac{\Delta T}{\Delta p_v} \quad \text{(4-23)}$$

$$\gamma = \frac{c_p \cdot p_a}{0.622 \cdot L_v} \quad \text{(4-24)}$$

$\boldsymbol{p_v}$ (Pa) - partial pressure of water vapor, $\boldsymbol{L_v}$ $(\frac{J}{kg})$ - latent heat of vaporization, $\boldsymbol{\gamma}$ (kPa/K) - psychrometric constant.

The Bowen ratio can be estimated by measuring the differences in temperature ΔT and in specific humidity Δq at two levels in the surface layer (e.g. with a dry and wet-bulb thermometer).

- **Surface Energy Balance:** knowing the Bowen ratio, the radiative (Q^*) and the ground heat flux (Q_G), the **latent** (Q_E) and **sensible heat flux** (Q_H) can be computed:

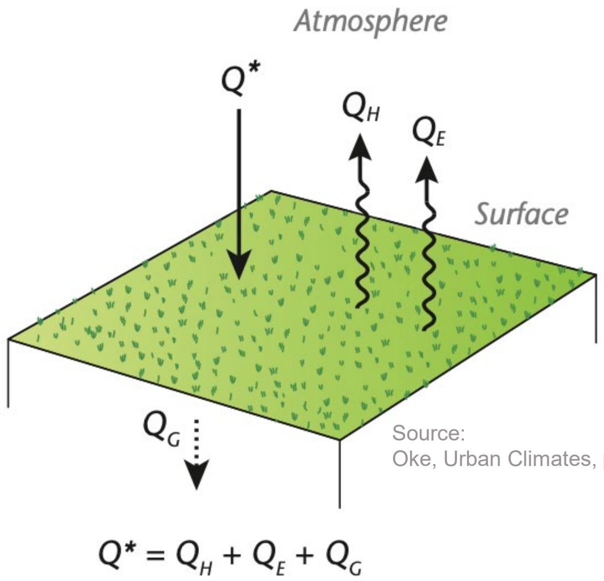
$$Q^* = Q_H + Q_E + Q_G \quad \text{(1-2)}$$

Latent heat flux

$$\boldsymbol{Q_E} = \frac{Q^* - Q_G}{1 + \boldsymbol{B}} \quad \text{(4-25)}$$

Sensible heat flux

$$\boldsymbol{Q_H} = \boldsymbol{B} \cdot \frac{Q^* - Q_G}{1 + \boldsymbol{B}} \quad \text{(4-26)}$$

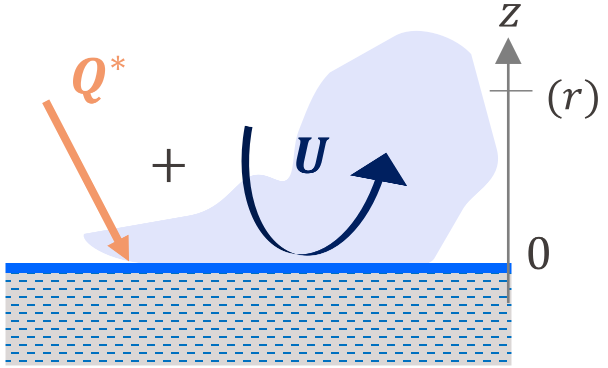


Source:
Oke, Urban Climates, p. 157

Evaporation rate (E): Penman method

- Assuming *water availability*, the **driving forces** of evaporation are **the supply of energy** (mainly from a radiative heat flux) and **the transport of mass away from the interface** *where evaporation happens* (i.e., the convection/advection of water vapor).

$$\boldsymbol{E_{pot}} = \underbrace{\frac{m}{m+\gamma}\cdot\left(\frac{Q^{*}-Q_G}{L_v}\right)}_{\text{Radiation term } \boldsymbol{E_S}} + \underbrace{\frac{\gamma}{m+\gamma}\cdot E_a}_{\text{Aerodynamic term } \boldsymbol{E_T}} \qquad \text{(4-27)}$$



- ○ $\boldsymbol{E_a}$ $(kg/m^2\cdot s)$ - drying power of air:

$$E_a = \rho\cdot C_W\cdot U\cdot(q_s - q_r) \qquad \text{(4-20b)}$$

q_s (kg/kg) - specific humidity at the reference height (r) assuming saturated air at T_r,
q_r (kg/kg) – actual specific humidity at the reference height (r),
C_W $(-)$ - bulk transfer coefficient for water vapor, U $(\frac{m}{s})$ – air speed at reference height

- ○ $\boldsymbol{m}$ (kPa/K) - slope of the saturated vapor pressure versus temperature curve at $(T_r + T_0)/2$:

$$m = \frac{p_{v,0} - p_{v,sat\_r}}{T_0 - T_r} \cong \frac{dp_{v,sat}}{dT} \qquad \text{(4-28a)}$$

$$m = 4098\cdot\frac{0.6108\cdot e^{\frac{17.27\cdot t_r}{t_r+237.3}}}{(t_r+237.3)^2} \qquad \text{(4-28b)}$$

- ○ $\boldsymbol{\gamma}$ (kPa/K) - psychrometric constant:

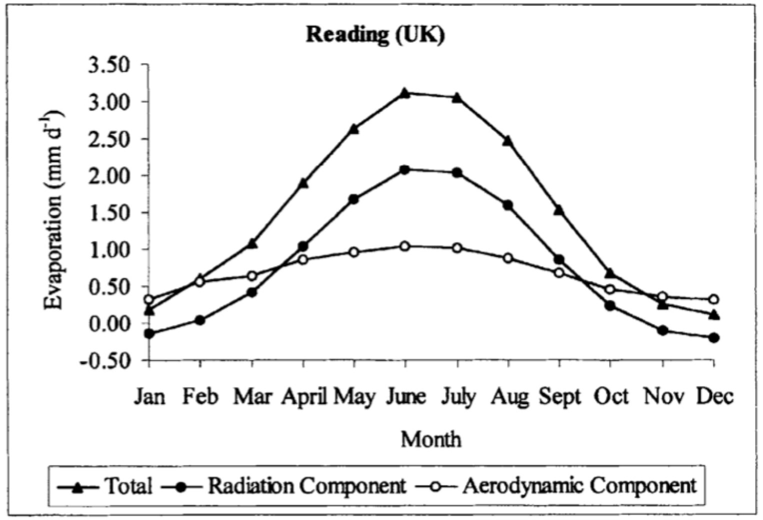
$$\gamma = \frac{c_p\cdot p_a}{0.622\cdot L_v} \qquad \text{(4-24)}$$

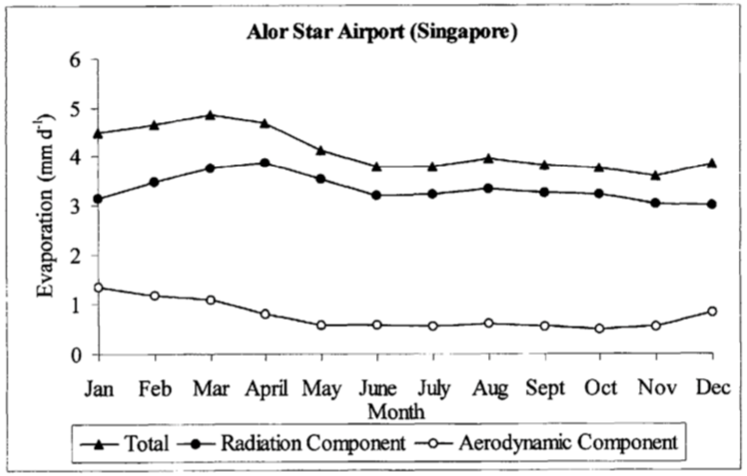
Evaporation rate (E): Penman method

- It is *the most widely used method* to calculate **daily average <u>potential</u> evaporation**.
- The **Penman** method is *a combination* of **the energy balance** and **the bulk transfer** methods:
  - **The radiation term E_S comes** from the **energy balance** method. The ground heat flux is *neglected* for the assessment of *daily evaporation* (see Lect. L6).
  - **The aerodynamic term E_T comes** from the **bulk transfer** method. It depends on the temperature, humidity of the air and on its wind speed.
  - **The balance between both terms** depends on the **climatic conditions**:
    - *For hot dry air*, the aerodynamic term is dominant
    - *For cold and humid air*, evaporation is less and there is more balance between both terms
- The **Bowen ratio** associated with **the Penman equation**:

$$B = \frac{\gamma}{m}\left(\frac{E_{pot} - E_a}{E_{pot}}\right) \qquad (4\text{-}29)$$

Monthly evaporation rates for various locations:





Source: Mason, Introduction to environmental physics, p. 355

Thank you for your attention

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